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Density matrix approach of the excitation on coherent states of the pseudoharmonic oscillator

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Abstract – We have examined the excitation on coherent states of the pseudoharmonic oscillator which are obtained by repeated action of the raising operator on the usual coherent states. By using the density matrix approach, we have examined some interesting properties (including the nonclassicality) of these states, both in pure and also in mixed (thermal) cases.

Introduction. – The excitation on coherent states (ECSs) can be considered as one of the possible generalizations of coherent states (CSs). These states may be useful in optical communications field which employed the nonclassical signal beams, usually mixed with thermal noise. On the other hand statistical properties of the CSs are useful in quantum optics and quantum electronics. The aim of this paper is firstly to build the ECSs for the pseudoharmonic oscillator (PHO) which is an intermediate oscillator between the harmonic oscillator (HO) and more anharmonic ones. Secondly, we study some properties of the ECSs for the PHO (of the Klauder-Perelomov (KP) kind) and thirdly, the superposition of these states with thermal light. The paper is organised as follows: in the second section, the ECSs are obtained by repeatedly operating the raising operator K_+ of the $SU(1,1)$ group on a usually normalizated KP-CSs of the PHO and it is demonstrated that these states are really coherent states. At the end of this section we will calculate the behaviour of the Mandel parameter $Q_{z,k;m}$ and we will study their dependence on z for different values of parameter m (the number of added quanta), in order to analyze the statistical properties of the E(KP)CSs for the PHO. In the subsequent section, the thermal noise which may be considered as being a thermal light, governed by a thermal density operator with the canonical weight function, is examined and we will find the P-quasi-distribution function.

The effective potential of the PHO is [1]

$$V_J(r) = \frac{m\omega^2}{8} r_J^2 \left(\frac{r}{r_J} - \frac{r_J}{r}\right)^2 + \frac{m\omega^2}{4}(r_J^2 - r_0^2), \quad (1)$$

where the equilibrium distance r_J, depending on the rotational quantum number J and a parameter λ are

$$\begin{aligned} r_J &= \sqrt{\frac{2\hbar}{m\omega}\left(\lambda^2 - \frac{1}{4}\right)^{\frac{1}{2}}}, \\ \lambda &= \sqrt{\left(J + \frac{1}{2}\right)^2 + \left(\frac{m\omega}{2\hbar} r_0^2\right)^2} \end{aligned} \quad (2)$$

The natural dynamic group associated with the bounded states of the PHO is $SU(1,1)$ [1], whose discrete representations are given by

$$K^2|v,k\rangle = k(k-1)|v+1,k\rangle, \quad (3)$$

$$K_+|v,k\rangle = \sqrt{(v+1)(v+2k)}|v+1,k\rangle, \quad (4)$$

$$K_-|v,k\rangle = \sqrt{v(v+2k-1)}|v-1,k\rangle, \quad (5)$$

where v is the vibrational quantum number, the constant $k = \frac{1}{2}(\lambda+1)$ play the role of the Bargmann index, K_+ and K_- being the raising and the lowering operators of this group.

The Klauder-Perelomov coherent states (KP-CSs) of the PHO are obtained if the generalized displacement unitary operator $\exp(\alpha K_+ - \alpha^* K_-)$ on the lowest state of the quantum system $|v=0,k\rangle$ are applied [2]:

$$\begin{aligned} |z,k\rangle &= \exp(\alpha K_+ - \alpha^* K_-)|0,k\rangle = \\ &\exp(zK_+)\exp(\Gamma K_3)\exp(-z^* K_-)|0,k\rangle, \end{aligned} \quad (6)$$

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where $\alpha = -\frac{1}{2}\theta\exp(-i\varphi), z = \frac{\alpha}{|\alpha|}\tan|\alpha|, \Gamma = \ln(1-|z|^2)$ and where the group generator K_3 is

$$K_3 = \frac{1}{2}[K_-, K_+], \qquad K_3|v,k\rangle = (v+k)|v,k\rangle. \tag{7}$$

In terms of the basis vectors $|v,k\rangle$, the normalized KP-CSs of the PHO may be expanded as [3]

$$|z,k\rangle = (1-|z|)^2)^k \sum_{v=0}^{\infty} \frac{z^v}{\sqrt{\rho(v;k)}}|v,k\rangle, \tag{8}$$

where the structure constants are expressed through the Euler's gamma-functions:

$$\rho(v;k) = \Gamma(2k)\frac{\Gamma(v+1)}{\Gamma(v+2k)}. \tag{9}$$

Excitation on the KP-CSs of the PHO. – Theoretically, the ECSs of the PHO can be obtained by repeatedly operating the raising operator K_+ of the $SU(1,1)$ group on a usually normalized KP-CSs of the PHO:

$$|z,k;m\rangle \equiv N_k^{(m)} K_+^m |z,k\rangle. \tag{10}$$

So, the usual KP-CSs of the PHO is $|z,k\rangle \equiv |z,k;0\rangle$. In order to calculate the normalization constant $N_k^{(m)}$, from the condition $\langle z,k;m|z,k;m\rangle = 1$ it is useful to employ the counterpart actions of the group generators on the bra vectors $\langle v,k|$:

$$\langle v,k|K_+ = \sqrt{v(v+2k-1)}\langle v-1,k|, \tag{11}$$

$$\langle v,k|K_- = \sqrt{(v+1)(v+2k)}\langle v+1,k|, \tag{12}$$

which leads to the expression:

$$[N_k^{(m)}]^{-2} = (1-|z|^2)^{2k}\frac{1}{\Gamma(2k)}G_{22}^{12}\left(-|z|^2|\ldots;0\right). \tag{13}$$

Here there appear Meijer's G-functions $G_{22}^{12}(-|z|^2|, \ldots, l)$ [4].

In the following, for writing convenience, we will use the notations

$$G_{22}^{12}\left(-|z|^2\middle|\begin{matrix}-m, & 1-2k-m; \\ 0; & l\end{matrix}\right) \equiv G_{22}^{12}\left(-|z|^2|\ldots;l\right) \tag{14}$$

and

$$G_{22}^{20}\left(|z|^2\middle|\begin{matrix}; & m, & 2k-1+m \\ 0, & & 0;\end{matrix}\right) \equiv G_{22}^{20}(|z|^2|\ldots;) \tag{15}$$

with $l = 0,1,2,\ldots$.

Particularly, for $l = 0$:

$$\frac{1}{\Gamma(2k)}G_{22}^{12}\left(-|z|^2|\ldots;\,0\right) \equiv S_0^{(m)}(|z|^2) = \sum_{v=0}^{\infty}\frac{(|z|^2)^v}{\rho(v,k;m)}. \tag{16}$$

Finally, the expansion of the excitation on the Klauder-Perelomov coherent states E(KP)CSs for the PHO in terms of the basis vectors is

$$|z,k;m\rangle = \left[\frac{\Gamma(2k)}{G_{22}^{12}\left(-|z|^2|\ldots;0\right)}\right]^{\frac{1}{2}} \times \sum_{v=0}^{\infty}\frac{z^v}{\sqrt{\rho(v,k;m)}}|v+m,k\rangle. \tag{17}$$

Consequently, the new structure constants are

$$\rho(v,k;m) = \Gamma(2k)\frac{[\Gamma(v+1)]^2}{\Gamma(v+m+1)\Gamma(v+2k+m)}. \tag{18}$$

Now, we must demonstrate that our obtained E(KP)CSs, are really the coherent states *i.e.* that they accomplish the minimal Klauder's prescriptions [5]: the normalization, the unity operator resolution and the continuity in the complex z-variable.

The normalization condition we have already shown when we obtained the normalization constant $N_k^{(m)}$, while the unity operator resolution reads

$$\int \mathrm{d}\mu(z,k;m)|z,k;m\rangle\langle z,k;m| = 1. \tag{19}$$

Yet, our tasks is to find the expression of integration measure $\mathrm{d}\mu(z,k;m)$. Their structure must be of the following type:

$$\mathrm{d}\mu(z,k;m) = \frac{\mathrm{d}^2 z}{\pi}h(|z|^2,k;m) = \frac{1}{2}\frac{\mathrm{d}\varphi}{\pi}\mathrm{d}(|z|^2)h(|z|^2,k;m), \tag{20}$$

where $z = |z|\exp(i\varphi), |z| < 1$. Following a standard procedure [1,3,5] this problem can be reduced to the Hausdorff moment problem and the final expression becomes:

$$\mathrm{d}\mu(z,k;m) = \frac{\mathrm{d}^2 z}{\pi}G_{22}^{12}\left(-|z|^2|\ldots;0\right) \times G_{22}^{20}\left(-|z|^2|\ldots;\right). \tag{21}$$

Due to the properties of Meijer's G-functions [4], the kernel of the integration measure is a positive function, according to Klauder's prescriptions [5].

Finally, the continuity in the complex z-variable is also accomplished. This means by calculating the distance between two E(KP)CSs.

The most convenient distance for explicit calculations is the Hilbert-Schmidt distance [6]:

$$D_{z';z} \equiv D(|z',k;m\rangle\langle z',k;m|, |z,k;m\rangle\langle z,k;m|) = \sqrt{2}\left[1-|\langle z',k;m|z,k;m\rangle|^2\right]^{\frac{1}{2}}, \tag{22}$$

which, after straightforward calculations, leads to the zero limit: $\lim_{z'\to z} D_{z';z} \to 0$, since

$$|\langle z',k;m|z,k;m\rangle|^2 = \frac{G_{22}^{12}\left(-z'^*z|\ldots;0\right)G_{22}^{12}\left(-z'z^*|\ldots;0\right)}{G_{22}^{12}\left(-|z'|^2|\ldots;0\right)G_{22}^{12}\left(-|z|^2|\ldots;0\right)}. \tag{23}$$

So, our obtained E(KP)CSs are really coherent states.

Our obtained E(KP)CSs are related to nonlinear CSs introduced (for the HO-1D) in [8] and [9], (see, also, the excellent review article [10] or book [11]). Similarly, for the PHO these states can be defined as the right eigenstates of the generalized annihilation operator $\tilde{K}_- = K_- f(N)$, *i.e.* they are in fact the CSs of the Barut-Girardello kind (*not* of the Klauder-Perelomov kind, as our obtained E(KP)CSs):

$$\tilde{K_-}|z,k;m\rangle = z|z,k;m\rangle. \tag{24}$$

After some straightforward calculations (but which are not the subject of this paper) we observed that our defined E(KP)CSs $|z,k;m\rangle$ correspond to the following intensity-dependent function:

$$f(N) = \frac{N-m}{N(N+2k-1)}, \tag{25}$$

where N is the number operator, as we see below.

We will point out here that for all potentials (excluding the HO-1D) the Barut-Girardello CSs *are different* from the Klauder-Perelomov CSs, even if, as we have showed earlier, these two kinds of states can be connected through the suitable defined intensity-dependent function.

At the end of this section let us we point out that the corresponding normalized density operator for the pure E(KP)CSs (in fact, the E(KP)CSs-projector) is

$$\rho_0^{(m)} \equiv |z,k;m\rangle\langle z,k;m| = [N_k^{(m)}]^2 (K_+)^m |z,k\rangle\langle z,k|(K_-)^m. \tag{26}$$

On the other hand, the naverage value for an observable A in a pure E(KP)CS will be

$$\langle A\rangle_{z,k;m} \equiv \langle z,k;m|A|z,k;m\rangle = \frac{\Gamma(2k)}{G_{22}^{12}\left(-|z|^2|\ldots;0\right)} \sum_{v',v=0}^{\infty} \frac{(z^*)^{v'} z^v}{\sqrt{\rho(v',k;m)\rho(v,k;m)}} \times \langle v'+m,k|A|v+m,k\rangle. \tag{27}$$

In order to examine the photon (or, generally, the boson) number distribution of the E(KP)CSs, we pay our attention to the average values of the integer powers of the photon number operator N whose action on the basis vectors is

$$N^n|v+m,k\rangle = (v+m)^n|v+m,k\rangle. \tag{28}$$

The average values are

$$\langle N^n\rangle_{z,k;m} = \frac{\Gamma(2k)}{G_{22}^{12}\left(-|z|^2|\ldots;0\right)} \times \sum_{v=0}^{\infty} \frac{(|z|^2)^v}{\rho(v,k;m)} (v+m)^n = \frac{1}{S_0^{(m)}(|z|^2)} \sum_{l=0}^{n} \binom{n}{l} m^{n-l} S_l^{(m)}, \tag{29}$$

where

$$S_l^{(m)}(|z|^2) \equiv \sum_{v=0}^{\infty} \frac{(|z|^2)^v}{\rho(v,k;m)} v^l. \tag{30}$$

We immediately observe that (if we use the notation: $|z|^2 \equiv x$)

$$S_{l\geqslant 1}^{(m)}(x) = \left(x\frac{\mathrm{d}}{\mathrm{d}x}\right)^l S_0^{(m)} = \sum_{j=1}^{l} c_j^{(l)} x^j \left(\frac{\mathrm{d}}{\mathrm{d}x}\right)^j S_0^{(m)}. \tag{31}$$

Generally, the positive coefficients $c_j^{(l)}$ can be obtained from the equality, according to our previous ansatz (appendix B, [7]):

$$v^l = \sum_{j=1}^{l} c_j^{(l)} \frac{v!}{(v-j)!}. \tag{32}$$

For $l=1$ and 2 these coefficients are equal to unity: $c_1^{(1)} = c_1^{(2)} = c_2^{(2)} = 1$.

The last equation is useful in order to point out the following differentiation property of Meijer's G-functions [5]:

$$x^j \left(\frac{\mathrm{d}}{\mathrm{d}x}\right)^j G_{22}^{12}\left(x\left|\begin{matrix} -m, & 1-2k-m; \\ 0; & 0 \end{matrix}\right.\right) = G_{33}^{13}\left(x\left|\begin{matrix} 0,- & m, & 1-2k-m; \\ & 0; & 0,j \end{matrix}\right.\right) = G_{22}^{12}\left(x\left|\begin{matrix} -m, & 1-2k-m; \\ 0; & j \end{matrix}\right.\right). \tag{33}$$

So, finally, we have

$$x^j \left(\frac{\mathrm{d}}{\mathrm{d}x}\right)^j G_{22}^{12}\left(x|\ldots,0\right) = G_{22}^{12}\left(x|\ldots,j\right), \tag{34}$$

$$\langle N^n\rangle_{z,k;m} = m^n + \sum_{l=1}^{n} \binom{n}{l} m^{n-l} \sum_{j=1}^{l} c_j^{(l)} \frac{G_{22}^{12}\left(-|z|^2|\ldots,j\right)}{G_{22}^{12}\left(-|z|^2|\ldots,0\right)}. \tag{35}$$

In order to examine the statistics of the E(KP)CSs for the PHO we will calculate the behaviour of the Mandel parameter $Q_{z,k;m}$ defined as

$$Q_{z,k;m} = \frac{\langle N^2\rangle_{z,k;m} - (\langle N\rangle_{z,k;m})^2}{\langle N\rangle_{z,k;m}} - 1. \tag{36}$$

This parameter measures the deviation of the variance of the photon distribution of a field state from the Poissonian distribution which characterize a certain "standard" state (*e.g.* the coherent state for the linear harmonic oscillator $|z\rangle = \exp(-\frac{1}{2}|z|^2)\sum_{v=0}^{\infty}\frac{z^v}{\sqrt{v!}}|n\rangle$). For the E(KP)CSs of the PHO, after straightforward calculation, we obtain

$$Q_{z,k;m} = \frac{\frac{G_{22}^{12}(-|z|^2|\ldots,2)}{G_{22}^{12}(-|z|^2|\ldots,0)} - \left[\frac{G_{22}^{12}(-|z|^2|\ldots,1)}{G_{22}^{12}(-|z|^2|\ldots,0)}\right]^2 - m}{m + \frac{G_{22}^{12}(-|z|^2|\ldots,1)}{G_{22}^{12}(-|z|^2|\ldots,0)}}. \tag{37}$$

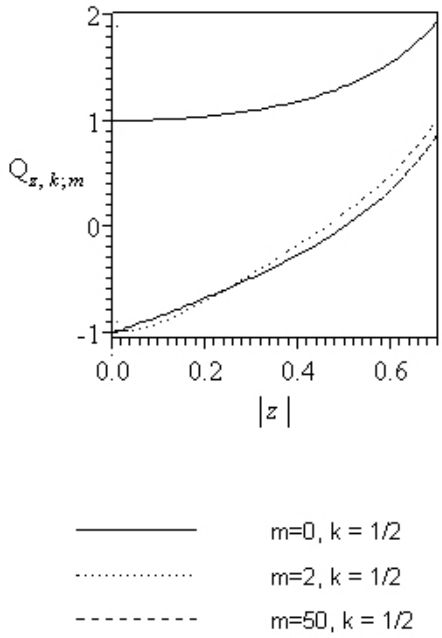

Fig. 1: Dependence of the Mandel parameter on $|z|$ with m as the parameter.

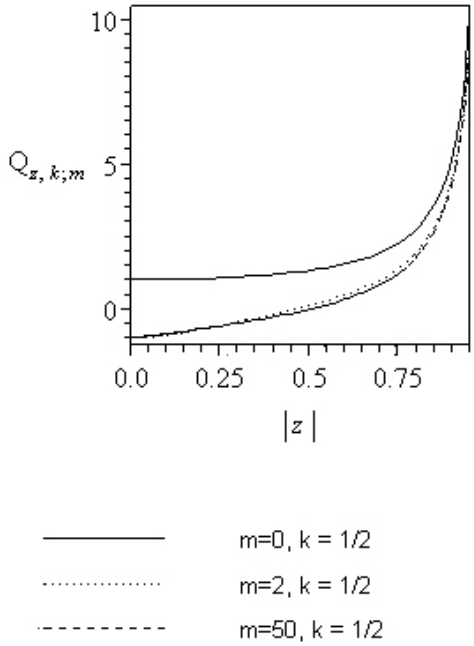

Fig. 2: Dependence of the Mandel parameter on $|z|$ with m as the parameter, by evincing the boundary behaviour at $|z| \to 1$.

Particularly, for $m=0$, we obtain the result for the Mandel parameter of usual KP-CSs $|z,k;0\rangle$:

$$Q_{z,k;0} = -\frac{|z|^2}{1-|z|^2} < 0. \tag{38}$$

Because $|z|<1$, it means that the states $|z,k;0\rangle$ involves according to the sub-Poissonian photon statistics (anti-bunching effect). On the other hand, in this later case the Mandel parameter are not dependent on the Bargmann index k. In fig. 1 we have represented the dependence of the Mandel parameter $Q_{z,k;m}$ as a function of $|z|<1$, where the number of added quanta (photons) m is considered as a parameter. We observe that only for the "nonadded" case (*i.e.* for $m=0$) the states are supra-Poissonian, *i.e.* $Q_{z,k;m}>0$ for any $|z|$. These states belong to the supra-Poissonian distribution and it is found to have variance $\Delta N \equiv \langle N^2\rangle - (\langle N\rangle)^2$ greater than that for a Poisson distribution. If m increases, for some small values of $|z|$, the states $|z,k;m\rangle$ become sub-Poissonian (with $Q_{z,k;m}<0$) and evidently for a certain value of $|z|$ they become Poissonian. If the number of added quanta m increases, consequently also increases the domain of nonclassicality. For all values of m the states $|z,k;m\rangle$ become strong classical for the limit $|z|\to 1$ (fig. 2).

The influence of thermal noise. – It is well known that the propagation of real signal beams is affected by the interaction with thermal noise. Theoretically, the thermal noise may be considered as being a thermal light, governed by a thermal density operator with the canonical weight function, *i.e.* by the density operator of single-mode thermal-states PHOs, in thermal equilibrium, characterized by the Hamiltonian H with eigenvalues [1]:

$$E_{vJ} = \hbar\omega\left(v+\frac{1}{2}\right) + \frac{\hbar\omega}{2}(2k-1) - \frac{m\omega^2}{4}r_0^2. \tag{39}$$

For the photon added (excited) Fock basis, the corresponding density operator is

$$\rho_{th}^{(m)} = \frac{1}{\mathrm{Tr}\rho_{th}^{(m)}} \sum_{J,v=0}^{\infty} \sum_{M=-J}^{J} e^{-\beta E_{v+m,J}} |v+m,k\rangle\langle v+m,k|, \tag{40}$$

where M is the quantum number of the projection of angular moment vector on a certain specific axis. After some simple calculations we obtained the following density operator of a thermal state with m added photons:

$$\rho_{th}^{(m)} = (1-X)\sum_{v=0}^{\infty} X^v |v+m,k\rangle\langle v+m,k|, \tag{41}$$

i.e. the same as that corresponding to the vibrational motion of the system of linear harmonic oscillators. We have used the notation: $X \equiv e^{-\beta\hbar\omega}$, $\beta = \frac{1}{k_B T}$.

In the following, we will expand the density operator of a thermal state in terms of projection operators onto the E(KP)CSs of the PHO, so that

$$\rho_{th}^{(m)} = \int \mathrm{d}\mu(z,k;m)|z,k;m\rangle P(|z|^2,k;m)\langle z,k;m| \tag{42}$$

and we try to find the P-quasi-distribution function. By substituting the expressions for the E(KP)CSs and the

integration measure, this problem is reduced to solving the Hausdorff moment problem [7], which finally leads to the expression

$$P(|z|^2,k;m)=\frac{1-X}{X}\frac{G_{22}^{20}\left(X^{-1}|z|^2|\ldots;\right)}{G_{22}^{20}\left(|z|^2|\ldots;\right)}. \tag{43}$$

Using the integrals of Meijer's functions product, it can be verified that the following equation holds:

$$\int \mathrm{d}\mu(z,k;m)P(|z|^2,k;m)=1. \tag{44}$$

For $m=0$ we recovered the corresponding expression of usual (nonexcited) KPCSs [1].

Moreover, the thermal average of an observable A, in the E(KP)CSs representation is

$$\langle A\rangle^{(m)}=\mathrm{Tr}(\rho_{th}^{(m)}A)= \int \mathrm{d}\mu(z,k;m)P(|z|^2,k;m)\langle A\rangle_{z,k;m}, \tag{45}$$

in which it is useful to know the averages in a pure E(KP)CSs, *i.e.* $\langle A\rangle_{z,k;m}$. The thermal average of an integer power of the photon number operators is then

$$\langle N^n\rangle^{(m)}=\int \mathrm{d}\mu(z,k;m)P(|z|^2,k;m)\langle N^n\rangle_{z,k;m}. \tag{46}$$

After a lengthy but straightforward calculation, the final result is

$$\langle N^n\rangle^{(m)}=m^n+\sum_{l=1}^{n}\binom{n}{l}m^{n-l}\sum_{j=1}^{l}c_j^{(l)}j!(\bar{n}_T)^j, \tag{47}$$

where we have used the expression of the main number of thermal photons (the Bose distribution function):

$$\bar{n}_T=\frac{X}{1-X}=\frac{1}{e^{\beta\hbar\omega}-1}. \tag{48}$$

Consequently, the thermal counterpart of the Mandel parameter (called the thermal Mandel parameter [7]) is

$$Q^{(m)}=\frac{\langle N^2\rangle^{(m)}-(\langle N\rangle^{(m)})^2}{\langle N\rangle^{(m)}}-1= \frac{(\bar{n}_T)^2-m}{\bar{n}_T+m}. \tag{49}$$

We quickly observe the following limits:

$$\lim_{T\to 0}\bar{n}_T=0, \qquad \lim_{T\to 0}Q^{(m)}=-1, \tag{50}$$

$$\lim_{T\to 0}\bar{n}_T=\infty, \qquad \lim_{T\to 0}Q^{(m)}=\infty. \tag{51}$$

Their dependence on T, with the number of added (excited) quanta m as a parameter is presented in figs. 3, 4. In fig. 3 we have represented the quantum Mandel parameter $Q^{(m)}$ (*i.e.* the counterpart of the Mandel parameter for a pure ECS $|z,k;m\rangle$) as a function

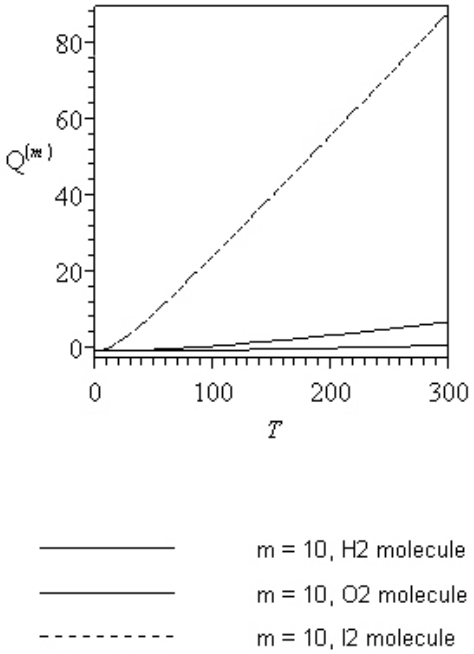

Fig. 3: Dependence of the quantum Mandel parameter on temperature with m as the parameter.

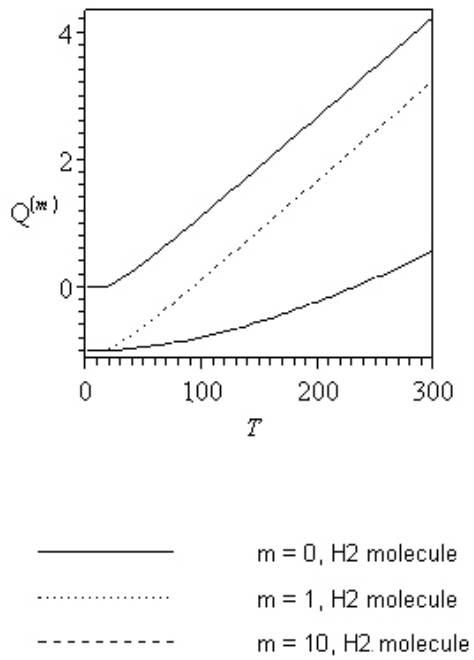

Fig. 4: Dependence of the quantum Mandel parameter on temperature, for the H_2 molecule, with m as the parameter.

of temperature T for different diatomic molecules (by considering that a diatomic molecule is an appropriate realization of the PHO model). We see that a lightly molecule (H_2) has a greater domain of temperature in which the thermal excited state is considered as being nonclassical (with $Q^{(m)}<0$), by comparing it to a hard molecule (I_2). Generally the conditions for different

Table 1: Conditions for realization of different kinds of distributions or of statistics.

Thermal Mandel parameter $Q^{(m)} = \frac{\bar{n}_T^2 - m}{\bar{n}_T + m}$	Distribution or statistics	Temperature T, $T_E = \frac{\hbar\omega}{k_T}$
$Q^{(m)} < 0$, $\bar{n}_T < \sqrt{m}$	sub-Poissonian	$T < \frac{T_E}{\ln\left(1+\frac{1}{\sqrt{m}}\right)}$
$Q^{(m)} = 0$, $\bar{n}_T = \sqrt{m}$	Poissonian	$T = \frac{T_E}{\ln\left(1+\frac{1}{\sqrt{m}}\right)}$
$Q^{(m)} > 0$, $\bar{n}_T > \sqrt{m}$	sub-Poissonian	$T > \frac{T_E}{\ln\left(1+\frac{1}{\sqrt{m}}\right)}$

kind of distributions or of statistics are presented in the table 1.

These aspects can also be relevant by examining fig. 4 where we have represented the dependence of the thermal Mandel parameter $Q^{(m)}$ as a function of temperature, the number of added quanta (photons) acting as parameter.

We see that the parameter $Q^{(m)}$ decreases with the increasing of m and consequently the domain of nonclassical character of thermal states becomes greater.

The thermal Mandel parameter becomes negative if the Bose distribution function $\bar{n}_T$ is smaller than $\sqrt{m}$ which correspond to a temperature $T < \frac{T_E}{\ln(1+\frac{1}{\sqrt{m}})}$, where $T_E = \frac{\hbar\omega}{k_B}$ is the Einstein-Debye characteristic temperature.

Concluding remarks. – In the present paper we have built the excitation on the coherent states of the PHO, by applying the raising operator K_+ on the usual KP-CS for the PHO. The succesive m-order application of this operator has as consequence the generation of a excited state $|z, k; m\rangle$. We have demonstrated that these states fulfill all the conditions required for a coherent state (the so called Klauder's minimal prescriptions), *i.e.* the normalization, the nonorthogonality, the continuity on the complex variable and the identity operator resolution.

In a significant part of our paper we have paid our attention to the examination of classical properties of these states. In this sense we have examined the behaviour of the Mandel parameter $Q_{z,k;m}$ as a function of variable $|z|$, the number of added (or excited) quanta m playing the role of a parameter. The criterion of nonclassical nature of these states is based on the sub-Poissonian statistics [11].

We have extended the examination of the statistical properties by examining the mixed (thermal) states described by the density operator ρ. In this sense we have used the previously defined thermal counterpart of the Mandel parameter. This allow the development of a quantitative criterion to characterize the nonclasssical properties of the field on the domains of temperature where the thermal Mandel parameter becomes negative.

In conclusion, we have introduced a new class of states that are generated by succesive action of the raising operator on the Klauder-Perelomov coherent states of the pseudoharmonic oscillator and we have shown the important nonclassical properties such states possess.

Generally, the E(KP)CSs belong to the class of photon (or boson)-added coherent states which is an interesting class of nonclassical states, as it is pointed out in [10,11].

To our knowledge, the construction of the E(KP)CSs for the PHO, including also the thermal Mandel parameter has not been previously derived in the literature.

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